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Determination of Gravity at Apollo 14 Landing Site

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Introduction

DURING the flight of Apollo 14, a measurement of the lunar gravitational field was made at the LM landing site (Fra Mauro). Due to electrical power constraints, the experiment was of short duration and did not interfere with the crew's preparations for the first extra vehicular activity (EVA) period. The measurement utilized the X-accelerometer of the LM Primary Guidance Navigation and Control System (PGNCS).[1] Essential to the experiment was a knowledge of the orientation of the LM navigation base (NB) with respect to the local gravity vector **g**. This information was obtained from the Lunar Surface Alignment Program which preceded the experiment. This Note describes how the information was used to determine g and how the value so obtained compares with the values obtained from four lunar gravity models.

Equipment

The apparatus employed to measure g consisted of two standard pieces of Apollo equipment: the LM Inertial Measurement Unit (IMU), and the LM Guidance Computer (LGC).[1] The IMU is a gimballed, three degree-of-freedom, gyroscopically stabilized device. Of interest here are two sets of coordinate axes associated with the IMU. The first is the LM NB with unit vectors $\hat{X}_{NB}$, $\hat{Y}_{NB}$, and $\hat{Z}_{NB}$. This system is defined with respect to the LM structural body. The second coordinate system is that of the IMU Stable Member. This system is defined by the orientation of three orthogonally mounted accelerometers. Let $\hat{X}_{SM}$, $\hat{Y}_{SM}$, and $\hat{Z}_{SM}$ represent the unit vectors of this coordinate system. The orientation of the NB with respect to the stable member is uniquely specified by three angles: outer gimbal angle (OGA); inner gimbal angle (IGA); middle gimbal angle (MGA).

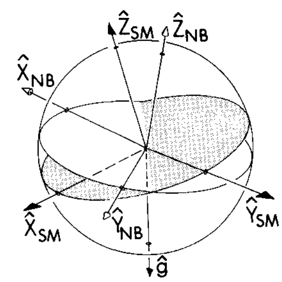

Fig. 1 Coordinate axes pertinent to the gravity experiment.

More specifically, the sensor of the gravitational acceleration was a Pulsed Integrating Pendulum Accelerometer (PIPA).[1] The equation governing the output of a PIPA is

$$a_i = (1 + SFE_i)(\Delta N/\Delta T) - B_i \tag{1}$$

where: ΔN_i = the numer of pulses from the ith accelerometer during the time ΔT; B_i = the bias of the ith accelerometer (the acceleration indicated in a zero **g** environment); SFE_i = scale factor error of the ith accelerometer†; and a_i = the average acceleration sensed by the ith accelerometer during the time ΔT. Because the bias is acceleration history sensitive, the equipment was used in such a way that the uncertainty in the bias of the X-accelerometer was inconsequential.

Methodology

After determining the orientation of the LM NB with respect to **g**, the gimbal angles required to align the X-accelerometer parallel and antiparallel to $\hat{g}$ ($\hat{g}$ = **g**/g) were calculated. The geometry of the problem is as given in Fig. 1. It can be shown that, given the orientation of $-\hat{g}$ with respect to the NB, the gimbal angles required to align $\hat{X}_{SM}$ parallel to $\hat{g}$ (position one) are given by:

$$OGA_1 = 0.00° \tag{2a}$$

$$IGA_1 = +\sin^{-1}\gamma + 180.00° \tag{2b}$$

$$MGA_1 = -\sin^{-1}(\beta/\cos IGA) \tag{2c}$$

To position $\hat{X}_{SM}$ antiparallel to $\hat{g}$ (position two), $OGA_2 = OGA_1$, $MGA_2 = MGA_1$ and the inner gimbal angle is given by

$$IGA_2 = +\sin^{-1}\gamma \tag{3}$$

Here β and γ are the components of $-\hat{g}$ along $\hat{Y}_{NB}$ and $\hat{Z}_{NB}$, respectively.

The values of β and γ were obtained via telemetry during the alignment program. Then, teams at the Manned Spacecraft Center (MSC) and the Draper Laboratory used Eqs. (2) and (3) to determine the required gimbal angles. While the crew was engaged in EVA I preparation, the IMU was aligned

Received April 4, 1971; revision received May 12, 1971.

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† To convert the number of pulses from an accelerometer into a mean acceleration, $\Delta N_i/\Delta T$ is ideally multiplied by a scale factor (SF) of 1 cm/sec/pulse. However, in practice the true scale factor is not unity and is instead given by $SF = (1 + SFE)$ cm/sec/pulse, where SFE is a small correction expressed in parts per million (ppm).

(via uplink from MSC) such that the X-accelerometer was parallel to $\hat{g}$ for approximately 10 min. The process was then repeated with the X-accelerometer antiparallel to $\hat{g}$.

In position one we have [by Eq. (1)]

$$-g = g_X^{(1)} = [1 + SFE_X)(\Delta N_X^{(1)}/\Delta T^{(1)}] - B_X \quad (4)$$

where $\Delta N_X^{(1)} < 0$. For position two we have

$$g = g_X^{(2)} = [1 + SFE_X)(\Delta N_X^{(2)}/\Delta T^{(2)}] - B_x \quad (5)$$

with $\Delta N_X^{(2)} > 0$. Equations (4) and (5) can be combined to yield

$$g = \tfrac{1}{2}[\Delta N_X^{(2)}/\Delta T^{(2)} - \Delta N_X^{(1)}/\Delta T^{(1)}](1 + SFE_X) \quad (6)$$

Equation (6) was used to calculate g for this experiment.

PIPA data and the associated times were sent to earth via telemetry. Subsequent reduction of the stored data yielded the raw data for the determination of g.

Data, Results and Analysis

The Lunar Surface Alignment Program gave for β and γ the values of -0.12183 and -0.33971, respectively. From Eqs. (2) and (3) these values gave for the gimbal angles: $OGA_1 = OGA_2 = 0.00°$; $IGA_1 = +178.05°$; $IGA_2 = -1.95°$; $MGA_1 = MGA_2 = +7.00°$. These angles were passed to the Guidance Officer in Mission Control Center whose first command sequence placed the X-PIPA input axis in the down position within 0.016° of the desired position, resulting in a measurement of $g_X^{(1)} = -161.55$ cm/sec^2. Then the X-PIPA input axis was commanded to the up position with a final alignment accuracy of 0.012°, resulting in a measurement of $g_X^{(2)} = +164.06$ cm/sec^2. Compensating for the misalignment of the X-PIPA from the known direction of g did not effect the final value since the correction was less than 0.01 cm/sec^2. Using Eq. (6) and a value for SFE of -945 ppm, which was based on a linear regression fit of the preflight measurements, the value for g was determined to be $g = 162.65$ cm/sec^2.

The alignment accuracies quoted previously were determined from the gimbal angles and the known orientation of the NB with respect to $\hat{g}$.

Comparison with Models

Prior to the Apollo 14 mission, the value of g for the Fra Mauro landing site was calculated using four lunar potential models,[2] resulting in a mean value of 162.57 cm/sec^2 with an uncertainty of 0.04 cm/sec^2.

The predicted values of g at the landing site assumed a radius of 937.735 naut miles. This was arrived at from stereo photography and landmark tracking data obtained from previous lunar missions. From a series of experiments performed during Apollo 14, the radius of the landing site was later determined[2] to be 937.577 naut miles. This value changes the mean value of acceleration for the lunar potential models to 162.63 cm/sec^2.

The uncertainty associated with the gravity measurement is 0.02 cm/sec^2; hence, within experimental error, the measured value of g ($g = 162.65 \pm 0.02$ cm/sec^2) equals the mean value of g, $\langle g_m \rangle$, calculated from the four potential models ($\langle g_m \rangle = 162.63 \pm 0.04$ cm/sec^2) employing the corrected value for the radius.

Conclusion

The measured value of g is consistent with the value of g observed among four lunar potential models while the uncertainty in the measured value of g is less than that of the models.

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Impulse Trajectory Correction by the "Cassiopee" System

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Introduction

IN PREPARATION for an astronomical observation program using sounding rockets, an attitude control device named Cassiopee[1] has been developed in France over the past few years. It is usually used to set the nose cones, previously separated from the rockets, on the celestial targets studied in a predetermined sequence.

Most of the experiments planned within the framework of this program include some photography, thus making it essential to retrieve the scientific payloads. This in turn entails a special requirement for the nose cones to make an accurate splashdown—a demand all the more pressing because the Guyana Space Center (CSG), the site chosen for the experimental program, is a seaside launching base, and successful recovery hinges largely on quick access to the floating payload.

With no special provisions made (apart from initiation of rocket spin, and careful correction for wind), the standard deviation on the impact of the two-stage rocket Beridan which is capable of carrying 400 kg to an altitude of 340 km, as called for under the observation program, is 39 km in any one direction. This suggests the need to aim at a nominal impact point 100 km off the coast so that the probability of having to destroy the rocket by remote control in order to protect the mainland should not exceed 0.5%. The distance of the coast from the planned splashdown and the dispersion provide the data that determine the operational methods and facilities to be used for locating and recovering the payload.

One way of improving accuracy would be to equip the rocket with an auto-pilot as a means of reducing dispersion, thereby enabling the nominal impact point to be brought nearer the coast. Without the use of costly high-precision gyroscopes, there is little chance of the standard deviation

Presented as Paper 70-1378 at the AIAA 2nd Sounding Rocket Technology Conference, Williamsburg, Va., December 7–9, 1970; submitted January 8, 1971; revision received April 30, 1971.

Index Catagories: Sounding Rocket Systems; Launch Vehicle and Missile Trajectories Launch Vehicle and Missile Configurational Design; Uncontrolled Rocket and Missile Dynamics.

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